

Mobilizing histories in mathematics teacher education: memories, social practices, and discursive games

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Abstract In the first part of this paper, we share and elucidate the way we mobilize histories in some disciplines that are part of the undergraduate courses in mathematics teacher education offered by State University of Campinas and Federal University of Rio Grande do Norte in Brazil. This way of mobilization can be featured as a set of collective *indisciplinary* problematizations occurring in a series of student investigations. Mobilizing practices of mathematics culture are the object of these investigations. These practices are performed by different communities both constituted by and constituent of different human activities. In the second part of this paper, we will discuss our way of mobilizing histories, contrasting it with the theoretical perspective of expansive learning, just as it has been defended by Yrjö Engeström, in his article *Non scolae sed vitae discimus—towards overcoming the encapsulation of school learning*. We will also attempt to highlight the role which this researcher has attributed to history in his model of expansive learning, a perspective based on the current research on *activity theory*.

Keywords Mathematics teacher education · Histories · Social practices · Discursive memory games · Indisciplinary problematization

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1 Introduction

Our point of view concerning the mobilization of histories in mathematics teacher education is based on the independent works we have been developing through the years in some disciplines that are part of the undergraduate courses in mathematics teacher education offered by State University of Campinas and Federal University of Rio Grande do Norte.¹

Due to the similarity of these works—sometimes shared with colleagues from our other institutions,² but always refined consistent with our professional practices—we think they can all be theoretically related.

In that sense, this article has a dual purpose. First, we want to relate and discuss the way we mobilize histories in mathematics teacher education. Our report does not rely upon research that would “test” the absolute or relative “efficiency” of a particular way of mobilizing histories in teacher education. However, due to the depth of our educational experience—repeatedly lived, reflected, and transformed, based on varying degrees of student involvement—we shall not give an account of it here in the usual sense.

Actually, the reference to our experience only establishes an explanatory strategy to show and discuss our

¹ Here, we are referring to the disciplines: *School Mathematics Education I and II*, which are part of the undergraduate courses of mathematics teacher education at the State University of Campinas, as well as the disciplines: *Topics of History of Mathematics* and *Epistemological Foundations of Mathematics* in courses at the Federal University of Rio Grande do Norte.

² For one of us, the discussion and refinement of this experience, which is briefly reported in the reference (Miguel and Miorim 2005), was carried out in a composite way until 2005 with Maria Ângela Miorim, teacher of the Faculdade de Educação of UNICAMP.

point of view, theoretically, regarding the way we have been mobilizing histories in mathematics teacher education. For this reason, we will try to highlight the theoretical–philosophical dialogue we have established in recent years, with valuable literature drawn from the various fields of knowledge such as history, sociology, linguistics, and anthropology, among others.

Second, we will deepen the theoretical–philosophical assessment of our experience, contrasting it by means of a game of approaches and differentiations with the theoretical perspective of expansive learning, just as it has been defended by Yrjö Engeström (1991). Moreover, we will also attempt to highlight the role which this researcher has attributed to history in his model of expansive learning, a perspective the current research on *activity theory*.

2 Mobilizing histories in mathematics teacher education

In our work with mathematics teacher education, histories have been marshaled in four stages: (1) the participants' individual memories of the ways in which the mathematical culture³ has been mobilized in their school lives; (2) the posing of guiding questions in the investigation of mathematical culture in the context of school practices; (3) investigation of official teaching guidelines and textbooks designed for teaching in Brazilian schools; (4) the interviewing of mathematics teachers and students from Elementary and High Schools.

These histories, taken together, are seen as flashes of discursive memory games which mobilize social practices of mathematics culture, performed by different communities⁴ which constitute (and are constituent of) different human activities.

Such discursive games of alternative memories are produced by the participants based on investigative works, and as they constitute and assess such games comparatively and collectively, they also continue developing their identities as peripheral members of the community of mathematics teachers. Each game becomes an object of interdisciplinary problematization along with the participants who, as they get involved with and develop these games,

³ Based on Thompson (1990), we mean by *culture* every intentional act of symbolic mobilization of objects of any nature, by institutional subjects, i.e., by subjects that always act and interact under the conditioning of instituted norms. We observe, then, that this symbolic and dynamic conception of culture does not see it as a repository of whatever it means, but as a set of semiotic practices performed by all situated subjects, i.e., by institutional subjects.

⁴ In the second section of this paper, we will refer to such communities as *ethno-communities*.

adhering to their objectives and rules, they themselves constitute a *community of cultural problematization*.

Since the notions of *social practices*, *discursive games* and *interdisciplinary problematization* each play a crucial role in the mobilization of histories in teacher education, we shall subdivide this section into three parts. In the first, we shall clarify the meanings with which we are mobilizing the social practical expressions and discursive games.

In the second, we shall outline the meaning we have given here to the notion of interdisciplinary problematization, as well as the role it plays in teacher education.

Likewise, in the third part of this section, we shall characterize the ways we mobilize histories in teacher education, in each of the four stages previously mentioned.

2.1 Histories, social practices and discursive games

Several thinkers have used the expression *social practice*, highlighting, among them: Foucault, Bourdieu, Habermas, Lyotard, and Certeau.

According to Schatzki (1996, pp. 89–90),

at least three notions of practice are prominent in the current conjuncture. According to one, practicing is learning how or improving one's ability to do something by repeatedly working at and carrying it out. It is in this sense of practice that adults practice the piano (...) In a second sense, practice is a temporally unfolding and spatially dispersed nexus of doings and sayings. Examples are cooking practices, voting practices (...). To say that the doings and the sayings forming a practice constitute a nexus is to say that they are linked in certain ways. Three major avenues of linkage are involved: (1) through understandings, for example, of what to say and do; (2) through explicit rules, principles, precepts, and instructions; and (3) through what I will call "teleo-affective" structures embracing ends, projects, tasks, purposes, beliefs, emotions, and moods. (...) A third prominent notion of practice is that of performing an action or carrying out a practice of the second sort.

It is important to emphasize that we are not using the word practice as opposed to theory and, in that sense, neither use of practice means a thoughtless action, nor does theory mean an *action-less* thought (Wenger 1998). Also, the word practice is not being used here to refer to a place where we do something, a place where—as it is often said—"we get our hands dirty".

When we talk about social practices we talk about articulated and previously interpreted groups of actions. Not about any action or group of actions, but actions which, even when carried out by one single person, must be connected to different types of human activity placed in

time and space in order to be defined and interpreted. We can, in that way, talk about reading practices, written steps of calculations, practices of garbage gathering, etc. So, the practices thus conceived as a group of actions are not synonymous with activity, although they can be carried out in different contexts of human activity.

More precisely, the term *social practice* here means an intentional and coordinated group of actions that simultaneously mobilizes cultural objects, memory, affections, values and powers, generating in the people who carry out such actions the feeling, albeit diffuse, of belonging to a determined community. These actions are not chaotic or random precisely because we recognize in them cultural objects that have a history. This history is remembered only because the cultural objects which this practice mobilizes are still valued by at least one community which keeps this memory alive for a reason. In that sense, a social practice is cultural because it always mobilizes cultural objects. On the other hand, a social practice is social because, even when it is carried out by a single person, it is always linked to human activities previously developed by socially organized communities. Therefore, from now on, we will simply speak of practices alone without adjectives.

Besides, the way that we interpret and carry out practices in different contexts varies from person to person, not just in their purposes, values, reasons, desires and interpretive resources, but also in the conditioning these contexts impose upon the realization of these practices. This is why we say that a practice also mobilizes affections.

Furthermore, every time it is put into circulation, a practice establishes a game, which is not always explicit, of asymmetrical power relationships among the participants of the community, as well as in a heterogeneous and differential game of valuation or of resistances between the participants of this community concerning this practice. And not only knowledge produces power, but also power produces knowledge, as Foucault warned us, getting to the point of stating that a society without power relationships can only be an abstraction.

On the other hand, the expression *discursive games* is being used here as an analogue in meaning to the notion of Wittgenstein's *language games* (Wittgenstein 2006).

We think that the relation we establish between the theoretical constructs *activity* and *practice* could also be described in Wittgensteinian terms. Since an activity, when viewed as an autonomous and self-signifying *form of life* (Wittgenstein 2006), is always developed by a community which at the same time performs practices connected to this activity, it also produces language games which are only distinguishable from their own correlative practices by the type of symbolic forms it mobilizes.

At the same time, *practices* and *language games* can be seen as indistinguishable from a semiotic point of view,

given the fact that both can be conceived as cultural objects or *symbolic forms*, in Thompson's sense (Thompson 1990). And so, the meanings of a practice would be inconceivable outside of the language games constituted in an activity (Miguel and Vilela 2008).

However, a practice is not always an activity, once a "same" practice can be performed in different activities, taking on diverse significance in the function of the different purposes which orient its performance in different activities. We might, for instance, carry out the practice of clapping hands at a birthday party, to salute the birthday celebrant, or at the door of a house without a doorbell to call for its inhabitants, or inside a classroom to call the students' attention, or even applauding an artist at a concert, as well as in many other situations. And in each one of them, the practice of "clapping hands" has a different meaning. In an analogue way, a practice of spatial orientation can be performed, with different meanings, in nautical, agricultural, topographical and astronomical activities, among others, mobilizing different purposes, instruments, methods, etc.

2.2 The practice of *INdisciplinary* problematization

The way we have mobilized histories in teacher education has been guided by a procedure of collective *INdisciplinary problematization* of successive assessments carried out by the students.

The term *INdisciplinary* is used by the Brazilian linguist Moita Lopes (2006), with a meaning similar to that given by the current transgressive theories to the notion of *transgression* (Pennycook 2006; Hooks 1994). For him, *indiscipline* does not only mean an act of transcendence, but above all, an act of transgressing disciplinary boundaries.

More than a mere epistemological crossing of disciplinary lines, the difference he seems to establish between the transdisciplinarity and transgressivity characterizes itself, above all, by a qualitative rupture with the disciplinary "way of seeing", i.e., with the objectivist paradigm that has persistently oriented the ways of producing the so-called "scientific" knowledge, according to a concept of rationality seen as "bodyless, lacking an understanding of the heterogeneity, fragmentation and mutability of the social subject, understood as placed in a sociohistorical void, and without contemplating ethical and power issues" (Moita Lopes 2006, p. 27).

This way of practicing the *indiscipline* shows *family resemblances* with the notion of *transgression* in the way it has been mobilized by Pennycook:

"(...) in the domain of transgressive theories, I'm interested in relating the concepts of *translocation*, as a

way of thinking the interrelation of the local inside the global; *transculturalization*, as a way of thinking culture and the processes of cultural interaction that allow fluidity in relationships; *transmodality* as a way of thinking the use of language as it is placed within the multiple ways of semiotic diffusion; *transtextualization*, as a way of thinking signs pervading contexts; *translation*, as a way of thinking the meaning as an act of interpretation that crosses the boundaries of comprehension modes; *transformation* as a way of thinking the constant change towards all the ways of meaning and interpretation” (Pennycook 2006, pp. 76–77).

Our way of practicing *indiscipline* as *transgression* is also consonant with the transgressive appeal suggested in Foucault’s piece, in the sense of stimulating the destabilizing practice, decolonization or deconstruction of determined ways of thinking, not only with the purpose of giving visibility to other ways of thinking, but also of creating new schemes of politicizing in the fight for a “new anti-disciplinary right” (Moita Lopes 2006, p. 27; Pennycook 2006, p. 75; Foucault 2000, p. 190).

In our work with the teachers, we neither tried to establish a relationship of opposition between transgression and normativeness, and nor do we see in that a problem of a logical, epistemological, or political nature. That is because, for us, normativeness is a constitutive instance of any practice and, less than a threat, its recognition is a condition for the achievement of transgressive practices, as well as for the exercise of our freedom. And even when, for certain purposes, the normative is seen as oppressor and in need of being transgressed, such transgression does not lead us to a state of *non-normativeness*, but simply establishes, or tries to, other ways of normativeness.

The way we have practiced the *indiscipline* as *transgression* is also inspired in the Wittgensteinian notion of *philosophical therapy* and in the Derridian notion of *deconstruction* (Derrida 2004). In the first case, the approach is characterized by the style of Wittgenstein’s philosophy that does not seek to “solve problems”, but to juxtapose words with the intent of highlighting absurdities arising from favoring one meaning to the detriment of others. In the second case, the approach is characterized by the style of Derrida’s philosophy of deconstructing all kinds of language games centered in the binary opposition between the sensitive and the intelligible that always refer ultimately to a referential or transcendental meaning.

The use we are making of the term *indisciplinary* here does not mean to suggest that it should be understood as a synonym for non-disciplinary, neither when the word *discipline* is seen as a limited school field of knowledge or as a limited field of scientific-academic investigation, nor when seen as a group of guiding norms of thought and

behavior. In that sense, when we speak in *indisciplinary (or transgressive) problematization of practices*, we are referring to a methodological practice oriented toward a political ethic which is simultaneously open, non-dogmatic, destabilized, and deconstructive.

Such practice voluntarily transgresses the boundaries of cultural disciplines, established in order to be recognized as equally legitimate, from the point of view of the teaching action, activities and practices which, for whatever reason, did not reach the disciplinary statute.

The pedagogic legitimacy of this methodological transgression rests not only upon the point of view according to which all human activities are producers of culture, but also on the point of view that a practice, in the passage from one field of activity to another, unavoidably disconnects itself from its original normative conditions and becomes formatted according to the normative conditions of the new field of activity into which it has been mobilized in an equally idiosyncratic way. Thus, we could no longer say that, strictly speaking, we would be facing the *same* practice.

The concept of *education* that orients this methodological procedure is the collective capacity of a community to problematize practices interactively and *indisciplinarily*. In the problematization process, there is no identifiable difference between roles of participants. That is because it is up to all of them to ask and answer questions collectively; however, aware that such responses are always open to discussion because they are based on an ethics policy equally open to discussion.

2.3 The four steps of mobilizing histories

In a first phase, participants are constituted as *a community of memory*.⁵ In this phase, they involve themselves in

⁵ The concept of *communities of memory* was suggested by Peter Burke (2000), who coined it inspired by the concept of *interpretative communities* that had previously been created by the literary critic Stanley Fish to analyze conflicts generated by possible alternative interpretations of literary texts. According to Fish, the operations and mental strategies we conduct in an interpretative act would be conditioned by the institutions within which we are already inserted, being in its turn, anchored in a public system of intelligibility, because “we do not have free and autonomous readers in a relation of perceptive adequacy or inadequacy towards a text which is equally autonomous. On the contrary, we have readers whose consciousness is built by a series of conventional notions that, when set into function, will constitute a conventional object seen conventionally” (Fish 1993, p. 162). In its turn, the concept of *communities of memory* is thus characterized in Burke’s words: “Facing the multiplicity of social identities and the co-existence of opposing and alternative memories (...) it is useful to think in pluralistic terms about the uses of memory by different social groups, which may have different visions of what is important or memorable. (...) It is important to ask: who wants who to remember what and why? Whose registered or preserved version is it?” (Burke 2000, p. 84).

an individual activity of reminiscence about their schooling processes, aiming to investigate, above all, mobilizing school practices of mathematics culture with which they would have been involved in their school context. Since, after this first phase, the community-class should split into four groups that shall conduct investigations focusing on mobilizing school practices of specific cultural objects of Brazilian mathematical education—such as trigonometry, logarithms, geometry, functions, etc.—we request the participants that their reminiscences, above all, reflect such practices. This request is the only direction given during this individual practice of reminiscence.

This investigative work of individual reminiscence is preceded by a session of collective problematization whose objective is to identify the different social uses of remembering and forgetting practices when mobilized in different contexts, such as in historic investigation, in criminal investigation, in psychoanalysis, in neuroscience, in the evaluation and reconstitution of historical-cultural heritage, in literature, in the rescue of cultural identity, etc. During this session, we make it explicit to the participants that the use we would make of their reminiscences in our classes would have as a purpose the creation of an interpretative diagram of school practices of mathematics culture mobilization, reflecting the collective memory of the community-class.

After this first problematization section, the participants produce a set of texts that, when placed within the broader context of the collective investigation process, [the set] is seen as a collection of discursive games of alternative individual memories of the community-class members, about mobilizing school practices of mathematics culture in a determined time and place.

After oral presentation of some of these discursive memory games to the community-class, all the games are subject to more systematical analytic treatments made by four of the participants who are not allowed to communicate with one another during the process. This way, based on an analytical reading of all individual memory texts, these four participants create another four discursive games of alternative social memories of the community-class. Such texts are said to be analytical because they must display a categorization, characterized description and interpretation of the mobilizing school practices of mathematical culture which manifest itself in the group of memories of every community-class. For the production of these texts, guidance is given so that they identify, in the individual memories of the participants, very frequent or common practices, as well as less frequent or idiosyncratic practices, since the intended goal is to give visibility to all of them. In that sense, the texts must not eliminate contrasts, disagreements or contradictions which, eventually,

may manifest themselves in the individual memories, given that the intended goal is not to produce rationalized histories.

Other directions are given so that the descriptive categories of the individual memories also consider: objects of the mathematics culture mobilized by the teaching practices; teaching methods; instruments or didactic material mobilized by these practices; norms and methods of behavior control and of coexistence (repression instruments, punishment and ways of exercising power) mobilized by these practices; norms that dictate the work of teachers and students in class; evaluation practices of mathematics learning; etc.

The purpose of such analysis is to construct an initial interpretative picture of these school practices from the perspective of the community of memory itself which is conducting the investigation of such practices. From this moment on, the community-class itself creates and distributes the first written documents referring to the history of Brazilian mathematics education to all its members. Of course, these text documents are flashes of histories as lived by the members of that particular community-class. This version of history, solely based on the memory of the community-class as a research source, is neither seen by us as illegitimate nor as false or incomplete, but as a version of the history told from the perspective of those community-class members, based on their experiences and interpretative resources.

In a second phase, participants split into four thematic groups in order to pose guiding questions that shall provide continuity to the investigation on mobilizing school practices of mathematics culture. Now, however, such questions shall focus on specific school practices related to mathematical objects that constitute the investigation theme of each group. Such questions can be of any nature: pedagogic, historic, philosophic, epistemological, logic, sociologic, etc.

Next, the questions raised by those groups go through a session of collective problematization, during which they are debated with the purpose of being expanded, modified or excluded, aiming now at the expectations and interests of the community-class as a whole. Throughout this session, we play a very important role in trying to broaden the motivation and expectation horizons in relation to the investigations that shall be conducted. We do so by presenting surprises, provoking different thoughts, posing new challenges, exposing conflicts, raising new conjectures, new questions, etc.

After this problematization section, groups start arranging a new set of questions, now analyzed by the community-class. So that such questions can be investigated, we make available to the group not only specific bibliographical references related to each of the themes,

but also a set of Basic Problematization Units (BPU). Below, we present an example of BPU.

In the sixth century B.C., Eupalinos of Megara invented a method to build a subterranean aqueduct, in Samos, ordered by the tyrant Policrates. In order to build it, slaves coming from the Greek island of Lesbos had to dig through the Kastro Mount. The tunnel – which took about 20 years to be build – still exists and is about 1036 m long and 2 m wide. It was simultaneously dug from both ends, B and D, of the aqueduct, i.e., from two points on the mountain, at different altitudes. The mistake in the meeting of both excavations was almost 10 m apart, horizontally, and 2.5 m vertically. Actually, this is a small mistake, less than 1%. Based on the book “*On the Dioptra*”, written much later by Heron of Alexandria, we infer that, supposedly, Eupalinos would have proceeded the following way: he chose a B point, next to the first entrance of the tunnel; then he chose an E point, on the plain part of the terrain around the mount, from where the point B could be seen; using an instrument similar to the *dioptra*, he obtained the direction EF, perpendicular to the direction EB; through a set of other perpendicular and consecutive directions – FG, GH, HK and KL –, he established, in the direction KL, the point M of the terrain, obtained by the perpendicular DM to the segment KL, where D is the other opposite entrance point of the tunnel; he measured, directly on the terrain floor that surrounded the mountain, the distances BE, EF, FG, GH, HK, KM and MD; based on these measurements, he established the measures of the imaginary perpendicular lines DN and BN, passing through the inside of the mountain and based on them, he then determined the alpha direction to be followed by both digger teams.

As the example shows, a BPU is nothing more than a *discursive memory flash* which describes a situated practice⁶ in a determined field of human activity, and it would actually have been used to answer the necessary piece of a community of practice at some point in the development of that activity in history.

After an oral or written presentation of this discursive flash to the participants, some questions posed by the

community-class give rise to an open interdisciplinary problematization process of BPU.

Next, we pose some questions that we have raised—among others posed by the participants—in different problematization sessions of the above-mentioned BPU:

- The method supposedly used by Eupalinos, showed itself adequate to the construction of the aqueduct? Why?
- Describe, characterize and discuss the technological artifacts and knowledge (whether mathematical or not) that could have been mobilized by Eupalinos to build the Samos aqueduct.
- How do you characterize the topographic activities and practices in ancient Greece around the sixth century B.C.?
- Today, since we have techniques and technological tools available—such as theodolite, laser rays, GPS, etc.—as well as a constituted trigonometry, we know that the same problem can be solved in different ways. Propose new methods to solve it.
- Does it make sense proposing a generic solution to Eupalinos’ problem? Why? Are generic methods always better than local methods?
- How would you characterize current topographic activities and practices?
- Suppose that you know only the point B to start digging the aqueduct in one of the sides of the mountain and that it would not be necessary to establish the point D a priori. Solve the problem considering these new conditions.
- Search and explore, analytically, images available in printed books or other kinds of available media which illustrate instruments and methods produced by our ancestors to carry out direct or indirect measuring of distances and angles.
- Describe some of the ancient and modern instruments for measuring distances and angles in physical space. Explain how they are used and the mathematical basis by which they are used.
- Formulate and solve problems involving measurements of non-accessible distances and angles.
- Formulate and solve, using the topographic triangulation method, a surveying problem that involves an estimate of the area of a plot of land.
- There is not a consensus among historians if the *dioptra* was really available at the time when Eupalinos built his aqueduct. Heron of Alexandria dedicated a whole book—named “*On the Dioptra*”—about the construction and usage of the *dioptra* in surveying activities. In the website http://www.mlhanas.de/Greeks/Heron_Alexandria.htm, you can get an example by Heron on

⁶ Practices are said to be *situated* (Lave 1988) in the sense of always being conducted under multiple normative conditioning relative to the institutional context of human activity itself in which they are carried out, as well as the institutional contexts of other fields of human activity. For being conceived here as human institutions, time and space also constitute normative contexts that condition all human activities, but not in one way only, nor determinant or homogeneous, once the time is always the time of the activity itself.

how to use the *Dioptra* to build a tunnel through two opposite points in a mountain. Consulting this and other sources, describe the dioptra and supply information about how it could have been used as a topographic or surveying instrument. Also explain how it could have been used by Eupalinos to establish perpendicular directions to other known ones. Establish comparisons between a dioptra and a theodolite.

- Who was Policrates and why did he want to build an aqueduct? Who was Eupalinos and why he had been called to build the aqueduct?
- In book 3, Herodotus refers to Eupalinos' aqueduct: "I have talked so much about the Samians, because, of all the Greeks, they have made the three greatest works of construction. One is a double-mouthed channel driven underground through a hill nine hundred feet high... The second is a mole in the sea around the harbor, one hundred and twenty feet deep. The length of the mole is a quarter of a mile. The third work of the Samians is the greatest temple that I have ever seen". This passage suggests that social practices related to civil construction and architecture activities were already well developed among ancient Greeks. Create a story in which the image of Eupalinos appears as a member of a community of practice linked to construction and architecture, around the sixth century B.C. In this story, try to characterize this community of practice concerning: cultural objects that it could have mobilized; the way the labor division was done among its members; the underlying rules guiding interpersonal relations which assured the maintenance of this kind of labor division; the power relations in the core of this community.
- In 1921, at the request of *Architectures* magazine, the book *Eupalinos or the Architect* was published. It contributed to the acclaim of the French writer and poet Paul Valéry. Do a brief but detailed report on this book and bring it along to discuss it in class.
- If you were using a BPU similar to this one along with High School students, with what goals would you do it, and how would you conduct your class to achieve such goals? Is this BPU accessible to High School students? Why?
- In what ways do you think this BPU could contribute to making school practices to mobilize, in an organic way, mathematical, scientific, technological, educational, artistic-literary and historical cultures? Why?

It is important to establish similarities and differences between a BPU and an interdisciplinary *problematization* procedure. Strictly speaking, a BPU is not a methodological tool, but a *discursive memory flash* that describes a practice (in the example considered here, the practice of building aqueducts) in a field of human activity (in the

example considered here, the activity of constructing public buildings) that, for some reason, is elected as a interdisciplinary *problematization* object. Such practice may, at some moment of its *problematization*, disconnect itself from the human activity field in which it was being initially *problematized* to connect to another activity field (as for instance: the literary activity field, the activity field of educators' formation, the school teaching activity field, etc.). On the other hand, the very procedure of interdisciplinary *problematization* can also be seen as a methodological practice that may be applied in the activity field of teacher education or in the field of school teaching activity. Thus, conceived, an interdisciplinary *problematization* also starts to be seen as a practice which is based on a *discursive memory flash* of another practice taken as a basic unit of a methodological *problematization*.

Often, the set of BPU we propose to the community-class cannot be found in textbooks or in official proposals related to mathematical school education. The set of BPU is produced in order to *problematize* mobilizing school practices of mathematics culture, contrasting them with ways that mathematics culture could have been (or has been) mobilized in other human activities. This does not mean, though, that these BPUs cannot be changed and used for other purposes, especially, with the High School or Elementary School. As a result, many of the BPU do not demand in-depth mathematical knowledge to be initially discussed in class, although the *problematization* of a BPU, due to its always open interdisciplinary nature, can achieve unpredictable levels of depth, sophistication, complexity, subtlety and originality.

Seen from another perspective, a BPU could also be considered a *discursive game mediator* of teacher education. In BPU production, we make an effort to value elements that are usually considered superfluous or irrelevant by mobilizing school practices of mathematics culture: contexts, historicity, informality and simplicity. The ranking of BPU is normally done according to two basic criteria: the nature of the activity fields that have probably motivated the creation, and the qualitative transformations of the mathematical objects that are being investigated, along with the chronological criteria that order these qualitative transformations. The chronological period involved is that of Pre-History up to the twenty-first century, and the involved practices are, e.g., the ones which have participated on the mobilization of mathematical objects in focus throughout history.

When we propose that participants explore BPU, we normally warn them not to see BPUs as if they were a conventional list of school or academic exercises, but as an invitation to *problematization*.

In our work, we have mainly explored practices connected to human activities, such as nautical, agricultural,

economical–financial, commercial, topographical, astrological–astronomical, mystical–religious, political, artistic, military, playful, educational, and scientific investigation.

In the second phase of our work, the purpose of the problematization is to create a set of oral and written discursive games based on the exploration of pertinent historical literature that supports the discussion of BPUs. These discursive games shall compose a second interpretative picture of mobilizing practices of mathematics culture, now with basis on the memories of different communities that could have performed such practices in the context of different human activities.

In a third phase, which is simultaneously developed with the second one, participants are involved in assessing official programs and textbooks written for teaching mathematics in Brazilian schools, from the nineteenth century through today. In this phase, the four groups of participants question these new document research sources based on the guiding questions which had been raised in the second phase. The dialogue established with these new sources is supported by the reading of a set of academic research texts about selected moments in the history of Brazilian school mathematics education. The results of these investigations constitute new discursive games of alternative memories of mobilizing social practices of mathematics culture. These games shall compose two new interpretative pictures of these practices by the participants. One offers the community perspective of official text producers written to reformulate and regulate mathematics education in Brazil at different times. The other game is in the community perspective of authors whose textbooks are destined for use in teaching school mathematics. These games are collectively presented and problematized by the community-class.

Finally, in a fourth and last phase, the four groups of participants carry out interviews with both mathematics teachers and Elementary and High School students. These interviews are formatted as printed texts and interpreted, having as background the reading of academic research works on the history of teachers' lives, from autobiographies and narratives. A new set of discursive memory games is produced, presented and collectively problematized by the community-class. These games shall compose a last interpretative picture of mobilizing school practices of mathematics culture, now in the perspective of the mathematics teaching community itself.

3 Mobilizing histories, what for? A dialogue with Engeström's theoretical perspective

In this section, we have as a goal to problematize how we mobilize histories in mathematics teacher education,

through a game of approximation and drawing back from the theoretical perspective of the expansive learning, such as it is presented by Engeström (1991), in his provocative article *Non scolae sed vitae discimus—towards overcoming the encapsulation of school learning*.

Even if Engeström's point of view—as well as the context in which his discussion is carried out—refers exclusively to the school teaching activity, we think there should exist, if not a convergence, at least an appropriate ethical–political consonance between the concept of education we defend when involved in the school teaching activity itself and that which we look forward to put into practice when—engaged in the activity of teachers education—we refer to the school teaching activity. Besides, although we believe a difference exists between the nature of the resource material and discursive resources available, as well as between the nature of the methodological procedures followed and the degree with which we mobilize them in one situation or another, we do not make an ethical–political distinction between the mobilization of histories within these two contexts of educational activity.

The provocative Latin aphorism present in the first part of the title of the Engeström's article—*We do not learn for school, but for life*—as well as his confident answer suggested in the subtitle, pointing to the possibility of overcoming the school's encapsulation, it urges us to question the contribution that our particular way to mobilize histories could bring to this discussion, comparatively to the role attributed to history by Engeström, in his expansive learning model.

According to this author,

since school is a historically formed practice, perhaps the initial step toward breaking its encapsulation is that students are invited to look at its contents and procedures critically, in the light of their history. Why not let the students themselves find out how their misconceptions are manufactured in school? (Engeström 1991, p. 254).

For him, among the original focuses of these misconceptions are the textbooks widely used as teaching tools that mediate school teaching and learning. In this sense, he questions himself:

if it is true that textbooks create closed and often illusory compartments in the minds of the students, should it not be desirous that students learn to treat textbooks as historical artifacts, as attempts to fix and crystallize certain generally accepted conceptions of the epoch? This would imply that (...) the students were led to (...) analyze and use the textbooks as limited sources often in need of thorough criticism (Engeström, 1991, p. 254).

We think that our particular way of mobilizing histories agrees with Engeström's purpose, not exactly "surpassing", but rather challenging school encapsulation based on history. It also supports his point of view about the need to make textbooks an object of systematic critical evaluation, due to their great power not exactly to create "misconceptions", but to naturalize several kinds of beliefs.

Particularly regarding the role of history in "overcoming" school encapsulation, Engeström's model of expansive learning adopts fundamental ideas developed by Davydov, as well as by Lave and Wenger.

According to Engeström, "Davydov's solution to the encapsulation of school learning is to spread school knowledge out into the world by making it dynamic and theoretically powerful when facing practical problems." (Engeström 1991, p. 251). Moreover, the solution he infers from Lave and Wenger's works to this same problem is "pushing communities of practice from the outside world into the school" (Engeström 1991, p. 253).

In Davydov's proposal putting the students into dialogue with the discoverers from the past, Engeström sees a great deal of advancement once such dialogue ends up, not only identifying the object of learning with the practical and historical context of the discovery, but also attributing a relative power to students, without pretending to eliminate the teacher's power (Engeström 1991, p. 188). Nevertheless, Engeström criticizes Davydov's strategy, considering it "*narrowly cognitive and scientific*" and not contributing to the modification of the social basis of school learning (Engeström 1991, p. 251).

He also sees a considerable advance from Lave and Wenger's proposal in relation to Davydov's, regarding the rupture of school learning encapsulation, once Lave and Wenger put the practical application context as the central object of school activity, and not the discovery context, as Davydov suggests. However, for him, Lave and Wenger's proposal fails because it does not systematically seek the genetic origin of the key ideas of school subjects so that they can be reproduced in the school learning process (1991, p. 190).

Even pointing out the limitations he finds in both of these proposals, Engeström believes that it is possible—and pedagogically convenient—to conceive school educational activity as a combination of the historical context of the discovery, such as proposed by Davydov, and the strategy of reproduction, in classes, of the context of application of school knowledge in non-educational social practices, such as proposed by Lave and Wenger.

However, as he thinks this combination would not be enough to overcome encapsulation of school learning, he proposes adding a third component to the object of the educational activity: the critical context. And he justifies such inclusion and combination based on the argument that

each one of the three complementary modes of knowing and learning suggested has distinct cognitive, motivational and social strengths. The context of the criticism highlights the powers of resisting, questioning, contradicting, and debating. The context of discovery highlights the powers of experimenting, modeling, symbolizing, and generalizing. The context of application highlights the powers of social relevance and embeddedness of knowledge, the community involvement and guided practice (...). In summary, expansive learning proposes to break the encapsulation of school learning, by expanding the object of learning (...). (Engeström 1991, p. 255; p. 256).

We think, however, that this strategy of additive expansion of the object of school learning activity cannot clearly identify the nature of this object, i.e., the nature of the analysis unit which school educational activity should take as the object of problematization. It seems clear that, for Engeström, this analysis unit would not be each one of the contexts (criticism, discovery or practical application) focused and/or isolated in succession. It would neither be a natural nor an isolated social phenomenon nor specific school subject content. Such an analysis unit seems to identify itself with any school content simultaneously focused on critical, discovery and practical application contexts. However, the starting point is still school content or isolated knowledge whose analysis would be gradually expanded until it reached meta-reflexive behaviors concerning the previous analytic phases themselves. It is especially this expansionist strategy of the object of educational activity that distinguishes Engeström's proposal from ours.

In our proposal, the objects set for problematization are discursive memory flashes produced by different communities of practice.

According to Engeström (1991, p. 256), in his expansive learning model, "school learning reflectively reorganizes itself as an activity system, and this kind of collective and reflective self-organization is becoming a necessity in practically all kinds of social practice". However, we think that problematizing histories of practices situated in different human activities are something that qualitatively and politically go beyond a "*reflective and collective self-organization*"—and even meta-reflexive—from the proper school learning activity itself.

It is also not about trying "*to push school knowledge out into the world*" neither "*to push communities of practice from the outside world into the school*". And if there is no doubt that we should align with the Latin aphorism that "*we do not learn for school, but for life*", Engeström's expansive learning proposal seems to have realized, only

partially, that *reproducing* or *simulating* life in school is something politically much more different than *problematizing*⁷ life in school.

We know today that no matter how some school practices try to move toward situated practices performed by other communities of practice, they always end up being formatted as typical school practices. Students usually see this and treat them this way as well. This happens because even trying to adapt them to the contemporary media style of “reality shows”, school practices will never be able to be effectively lived by students as non-educational practices in the same way that they are experienced by those who carry them out in other human activities.

This impossibility of effectively living school practices is because in going from one activity to another practice unavoidably get disconnected from their original normative conditioning and start being shaped according to the normative conditioning of the new activity in which they are always performed in an idiosyncratic way. Thus, we could no longer say that, strictly speaking, we would be facing the *same* practice. In this way, powers, values and affections mobilized by those practices in a certain activity field can also be considerably modified.

Walkerdine called our attention to these facts when she comparatively analyzed the *shopping game* performed in class, involving 7-year-old children, with selected shopping practices effectively performed in non-educational activities. It states that the group of children in a school situation,

has found, in the disconnection between the prices presented in the activity and the real prices, reason for fun and fantasy. Every time children, in a school situation, wanted to go shopping, they always had a new ten-cent coin, that is, their money never diminished, as it would happen if they shopped in real life. Besides, the object of their purchases was not a set of goods, but some calculations written on the paper. In other words, in the store simulated in the classroom, no actual exchanges would happen, but only simulated exchanges (Walkerdine 2004, p. 118).

But how should we interpret those calculi written on paper in the simulated school situation? At first sight, affirms Walkerdine (2004, p. 118), “we could think that the calculi would have been, actually, abstracted from everyday practices”. However, she continues, “the usage of the term “abstracted” can be deceiving, because new calculi exist as a discursive relation in a new set of practices, such as those from school math, with their ways of regulation and submission” (Walkerdine 2004, p. 118).

For her, what could have happened was not a passage from the concrete to the abstract, or from the abstract to the concrete, but “a passing from one discursive practice to another”. And yet, according to her, “what would have started to be valued, in the school situation, as a higher order activity would be the effort to regulate and control, through reason, the social order which has the bourgeois subject as a norm, the one who does not need to calculate to survive” (Walkerdine 2004, p. 118).

Besides, according to her, in the school situation each child positions himself as a subject in a different way from which it would happen in non-school practices. This way could be similar to or different from subjection patterns in which such children are involved in other practices, but according to Walkerdine (2004, p. 118), “evidence suggests that, to oppressed groups, patterns are substantially different, a fact which could bring unfolding, important affective and political developments”.

This simple example strongly suggests to us that we definitely need to break from the general assumption shared by structuralist–formalist psychological perspectives. This presupposition states that—in order to be supposedly more theoretical, more generic, more abstract, more structuring and more structured—school practices would have the power to transfer and apply themselves, in a responsible way, to non-educational fields of human activities. Therefore, Walkerdine’s example suggests that we question the belief in the conservation of a supposed original purity and essentiality of the practices in their different circulation process and devote special attention to the idiosyncratic purposes and effects of these processes in each field of human activity.

Walkerdine’s example calls into question curricular practices that insist upon structuring themselves in such generic, abstract and vague categories, like “school contents”, “historical knowledge gathered by the mankind”, etc. It also leads us to arrive at the conclusion that school is not similar to life, nor life to school; a ten or a zero in life are not like a 10 or a zero at school; that success or failure in life is not like success or failure at school; that what is explicit in life is not like what is explicit in the school’s *didactic contract*; and that the concealed in life is not like the *concealed curriculum* of school.

We think that this happens because school is not and could not be found in life itself, but at the edge of it, just like the Wittgensteinian subject is not in the world, but at the world’s edge. It is at this edge of life that we think the object of the educational activity can be found, and consequently, the object of the teacher education activity. And when placing this object at the edge of life, on the boundary that connects and, at the same time, separates an internal and an external life as it is, we mean that this object cannot be a reflection of life as it is, but as we wish it were.

⁷ Practices of problematization always mobilize values associated with political ethics. And there is no educational activity in the absence of these types of values.

4 Final considerations

Somehow, all human activities find itself encapsulated in itself, even if relationships with other human activities are established. The same happens with school educational activity. From our perspective, the object of mathematics teacher education—and, consequently, that of the school educational activity—consists of discursive memory flash games produced by different communities of practice, upon which the problematization practice falls. In this way, the practice of problematization becomes constituent of the ethnicity⁸ of the school ethno-community. This *ethnicity*, i.e., the cultural bond that links and identifies the members of this community, is the commitment with the promotion of practices of problematizing histories of different practices and is not exclusively considered “scientific.”

Thus, if on the one hand, the so-called scientific practices constitute themselves based on the suspicion of practices performed by other ethno-communities—deconstructing them as scientific and, consequently, constructing them as *lower practices* (Lave 1996)—these, on the other hand, have also resisted and persisted in their suspicion or indifference in relation to the so-called scientific practices. However, from our perspective, the scientific ethno-community itself should be seen as one among other ethno-communities, even if they do not see themselves this way. This means that, for the ethno-community educational school, the cultural scientific practices should not enjoy any previous epistemological privileges or political absolutes in relation to practices accomplished by other ethno-communities.

Thus, the nature of the commitment shared by the school teaching community has imposed the activity that its members develop, not exactly a scientific character, but a deconstructive ethical–political nature. This means that, based on a non-ethnocentric political ethic—but one that fights all forms of submission, discrimination and exploitation of man by man, school education should have as a purpose the preparation of people for the transgressive problematization of practices and discursive games constituting all forms of public life, i.e., of all the ways people have to organize themselves publicly in ethno-communities.

⁸ We conceive an ethnical subject as that one who characterizes himself as feeling as a member, simultaneously, of several practice communities. These multiple and diverse feelings of belonging constitute the dynamic and always open process of identification of an ethnical subject, i.e., of constitution at the same time delimited and illimitable of his identity. The prefix *ethno* is being used here in a wide sense. When applied to a community, it works as the characterizing element (but not defining) of its identity, which constitutes and changes throughout its historical process of inclusion in a determined activity which, in its turn, also changes due to the actions of this community.

In this political school teaching projects, conceived as a group of politicization practices, in which the school curriculum is to be organized—in a dynamic and investigative way—based on BPUs, both mathematics and histories are to be conceived not as fixed and distinguishable blocks of contents, but as distinct ways of seeing and investigating practices. In this context, mathematics should be conceived as practices of investigation of the normative aspects of different practices; similarly, histories are understood as practices of investigation of plural memories of different practices.

And because we think that there must be a teleological consonance between an ethical–political project for school teaching and an ethical–political project for teacher education, we also think that such consonance should be related to our ways of conceiving the role of the mobilization of histories in these two fields of teaching activity. For this interdisciplinary ethical–political project, it does not make sense to assign different roles to be played by histories, mathematics and other school subjects. Ultimately, all of them would fit the role of production of new forms of politicization, i.e., new ways of interfering in social relationships and in current ways of organizing different fields of human activity.

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